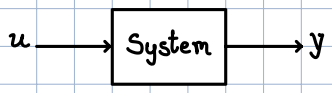
1. Recital 20.09.24

Modeling

Basic Intro:

We want to learn how to mathematically represent dynamic systems. The underlying concepts will be dealt with in detail in Mechanics III.

Initially we can think of a system as a black box that takes some input u and produces some output y.



Now we want to write down equations that express the output as a function of the input, and some internal parameters. Tipically, we describe the system as a system of ordinary differential equations.

In our case we usually look at the change of some storage quantity over time as a function of the flow of some other quantity.

$$\frac{d}{dt}\,\text{storage} = \sum \text{inflow} - \sum \text{outflow}$$

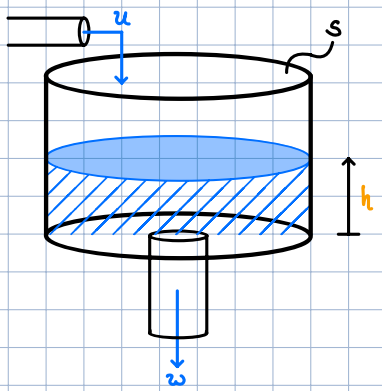
Example: Water tank

u : incoming flow $\left[\frac{m^3}{s}\right]$

w : outgoing flow $\left[\frac{m^3}{s}\right]$

h : water level $[m]$ (State)

S : cross-sectional area $[m^2]$



Here the output y is just the water level h. We can now model the system as follows:

$$S\frac{d}{dt}h(t) = u(t) - w(t)$$

$$y(t) = h(t)$$

In this system $h(t)$ summarizes everything that happend before some time t. It is called the state of the system. States are usually indicated with the letter x.